A Study on (α, β)-Level Subsets of Bipolar Valued Q-Fuzzy Subgroup

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Abstract: In this paper, we study on (α, β)-level subsets of Bipolar valued Q-fuzzy subgroup and prove some results on these.
Keywords: Fuzzy subset, Fuzzy subgroup, Bipolar valued Q-fuzzy set, Bipolar valued Q-fuzzy subgroup.

I. INTRODUCTION

In 1965, Zadeh [7] introduced the notion of a fuzzy subsets of a set, fuzzy sets are a kind of useful Mathematical structure to represent a collection of objects whose boundary is vaque. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as Intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets, etc. [2]. Lee [3] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0,1]$ to $[-1,1]$.

In a bipolar-valued fuzzy sets, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree $(0,1]$ indicates that elements somewhat satisfy the property and the membership degree $[-1.0)$ indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [3,4].

We introduce the concept of (α, β)-level subsets of Bipolar-valued Q-fuzzy subgroup.

II. PRELIMINARIES

Definition 2.1 [7]: Let X be a non-empty set. A fuzzy subset A of X is a function $A: X \rightarrow [0,1]$.

Definition 2.2 [1]: Let $(G, \cdot)$ be a group A fuzzy subset A of G is said to be a fuzzy subgroup of G if the following conditions are satisfied:

(i) $A(x, y) \geq \min\{A(x), A(y)\}$,
(ii) $A(x^{-1}) \geq A(x)$ for all x and y in G.

Definition 2.3 [3]: Let A and B be two bipolar valued fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subset B$ if and only if $A^{+}(x) \leq B^{+}(x)$ and $A^{-}(x) \geq B^{-}(x)$, for all $x \in X$.
(ii) $A = B$ if and only if $A^{+}(x) = B^{+}(x)$ and $A^{-}(x) = B^{-}(x)$, for all $x \in X$.
(iii) $A \cap B = \{(x, \min(A^{+}(x), B^{+}(x), \max(A^{-}(x), B^{-}(x)))\rangle / x \in X \}$
(iv) $A \cup B = \{(x, \max(A^{+}(x), B^{+}(x), \min(A^{-}(x), B^{-}(x)))\rangle / x \in X \}$

Definition 2.4 [6]: A bipolar – valued Q-fuzzy set (BVQFS) A in X is defined as an object of the form $A = \{((x, q), A^{+}(x, q), A^{-}(x, q)\rangle / x \in X$ and q in $Q\}$ where

$A^{+}: X \times Q \rightarrow [0,1]$ and $A^{-}: X \times Q \rightarrow [-1,0]$. The positive membership degree $A^{+}(x, q)$ denote the satisfaction degree of an element (x, q) to the property corresponding to a bipolar-valued Q-fuzzy set

A and the negative membership degree $A^{-}(x, q)$ denotes the satisfaction on degree of an element (x, q) to some implicit counter-property corresponding to a bipolar-valued Q-fuzzy set A.

If $A^{+}(x, q) \neq 0$ and $A^{-}(x, q) = 0$, it is the situation that (x, q) is regarded as having only positive satisfaction for A and if $A^{+}(x, q) = 0$ and $A^{-}(x, q) \neq 0$, it is the situation that (x, q) does not satisfy the property of A, but somewhat satisfies the counter property of A.

It is possible for an element (x, q) to be such that $A^{+}(x, q) \neq 0$ and $A^{-}(x, q) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

Example $A = \{\langle(a, q), 0.7, -0.4\rangle, \langle(b, q), 0.6, -0.7\rangle, \langle(c, q), 0.5, -0.8\rangle\}$ is a bipolar-valued Q-fuzzy subset of $X = \{a, b, c\}$, where $Q = \{q\}$.

Definition 2.5 [6]: Let G be a group and Q be a non-empty. A bipolar-valued Q-fuzzy subset A of G is said to be a bipolar-valued Q-fuzzy subgroup of G (BVQFSG) if the following conditions are satisfied;

(i) $A^{+}(xy, q) \geq \min\{A^{+}(x, q), A^{+}(y, q)\}$
(ii) $A^{+}(x^{-1}, q) \geq A^{+}(x, q)$
(iii) $A^{-}(xy, q) \leq \max\{A^{-}(x, q), A^{-}(y, q)\}$
(iv) $A^{-}(x^{-1}, q) \leq A^{-}(x, q)$ for all x and y in G and q in Q.

Example 2.6 [6]: Let $G = \{1, -1, i, -i\}$ be a group with respect to the ordinary multiplication and $Q = \{q\}$. Then $A = \{\langle(1, q), 0.5, -0.6\rangle, \langle(-1, q), 0.4, -0.5\rangle, \langle(i, q), 0.2, -0.4\rangle, \langle(-i, q), 0.2, -0.4\rangle\}$ is a bipolar-valued Q-fuzzy subgroup of G.

Definition 2.7 Let $(G,\cdot)$ be a group. A bipolar-valued Q-fuzzy subgroup A of G is said to be a bipolar-valued Q-fuzzy normal subgroup (BVQFNSG) of G if $A^{+}(xy, q) = A^{+}(yx, q)$ and $A^{-}(xy, q) = A^{-}(yx, q)$. For all x and y in G and q in Q.

Definition 2.8 Let G and $G^{'}$ be any two groups. Then the function $f: G \rightarrow G'$ is said to be an anti- homomorphism if $f(xy) = f(y)f(x)$, for all x and y in G.

Definition 2.9 Let X and X' be any two sets. Let $f: X \to X'$ be any function and let A be a bipolar- valued Q-fuzzy subset in X, V be a bipolar-valued Q-fuzzy subset in $f(x) = X'$, defined by $V^{+}(y,q) = \sup_{x \in f^{-1}(y)} A^{+}(x,q)$ and $V^{-}(y,q) = \inf_{x \in f^{-1}(y)} A^{-}(x,q)$, for all x in X and y in X' and q in Q. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 2.10 Let $A = \langle A^{+}, A^{-}\rangle$ be a bipolar-valued Q-fuzzy subset of X. Then a bipolar-valued Q- fuzzy subset $A^{0} = \langle A^{0+}, A^{0-}\rangle$ of X, is defined as $A^{0+}(x,q) = A^{+}(x,q) / A^{+}(e,q)$ and $A^{0-}(x,q) = A^{-}(x,q) / A^{-}(e,q)$ for all x in X and q in Q.

(α, β)-Level subsets of Bipolar valued Q-fuzzy subgroup

Definition 3.1 Let $A = \langle A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subset of G. For α in $[0,1]$ and β in $[-1,0]$, the (α,β)-level subset of A is the set $A_{(\alpha,\beta)} = \{x \in G; A^{+}(x,q) \geq \alpha \ \ and \ \ A^{-}(x,q) \leq \beta\}$.

Example 3.2 Consider the group $G = \{0,1,2,3,4\}$ and $Q = \{q\}$. Let

$A = \{((0,q), 0.5, -0.1), ((1,q), 0.4, -0.3), ((2,q), 0.6, -0.05), ((3,q), 0.45, -0.2), ((4,q), 0.2, -0.5)\}$

be a bipolar valued Q-fuzzy subset of G and $\alpha = 0.4, \beta = -0.1$. Then $(0.4, -0.1)$-level subset of A is $A_{(0.4,-0.1)} = \{0,1,3\}$.

Definition 3.3 Let $A = \langle A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subset of G. For α in $[0,1]$, the A^{+}-level of α-cut of A is the set $P(A^{+}, \alpha) = \{x \in G; A^{+}(x,q) \geq \alpha\}$

Example 3.4 Consider the group $G = \{0,1,2,3,4\}$ and $Q = \{q\}$. Let

$A = \{((0,q), 0.7, -0.1), ((1,q), 0.4, -0.3), ((2,q), 0.6, -0.5), ((3,q), 0.45, -0.2), ((4,q), 0.3, -0.5)\}$

be a bipolar valued Q-fuzzy subset of G and $\alpha = 0.4$. Then A^{+}-level 0.4-

cut of A is $P(A^{+}, 0.4) = \{0,1,2,3\}$

Definition 3.5 Let $A = \langle A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subset of G. For β in $[-1,0]$. The A^{-}- level β-cut of A is the set $N(A^{-}, \beta) = \{x \in G, A^{-}(x,q) \leq \beta\}$.

Example 3.6 Consider the group $G = \{0,1,2,3,4\}$ and $Q = \{q\}$.

$A = \{((0,q), 0.5, -0.1), ((1,q), 0.3, -0.3), ((2,q), 0.6, -0.05), ((3,q), 0.4, -0.2), ((4,q), 0.9, -0.5)\}$

be a bipolar valued Q-fuzzy subset of G and $\beta = -0.1$. Then A^{-}-level

-0.1-cut of A is $q(A^{-}, 0.1) = \{0,1,3,4\}$.

Definition 3.7 Let $A = \langle A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subset of G and α in

$[0, 1 - \sup\{A^{+}(x,q)\}]$, β in $[0, -1 - \inf\{A^{-}(x,q)\}]$, then $T = \langle T^{+}, T^{-}\rangle = T^{A}_{(\alpha,\beta)}$ is called a bipolar valued Q-fuzzy translation of A if

$T^{+}(x,q) = T^{+A}_{\alpha}(x,q) = A^{+}(x,q) + \alpha$, $T^{-}(x,q) = T^{-A}_{\beta}(x,q) = A^{-}(x,q) + \beta$

for all x in G and q in Q.

Theorem 3.8 Let $A = (A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. Then for α in $[0,1]$ and β in $[-1,0]$ such that $\alpha \leq A^{+}(e, q)$. $A(\alpha, \beta)$ is a (α, β)-level subgroup of G.

Proof: For all x, y in $A_{(\alpha,\beta)}$ and q in Q, we have $A^{+}(x, q) \leq \alpha$ and $A^{-}(x, q) \leq \beta$ and $A^{+}(y, q) \geq \alpha$ and $A^{-}(y, q) \leq \beta$. Now, $A^{+}(xy^{-1}, q) \geq \min\{A^{+}(x, q), A^{+}(y, q)\} \geq \min\{\alpha, \alpha\} = \alpha \Rightarrow A^{+}(xy^{-1}, q) \geq \alpha$

And $A^{-1}(xy^{-1}, q) \leq \max\{A^{-}(x, q), A^{-}(y, q)\} \leq \max\{\beta, \beta\} = \beta \Rightarrow A^{-}(xy^{-1}, q) \leq \beta$.

$\therefore xy^{-1}$ in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a (α, β)-level subgroup of G.

Theorem 3.9 Let $A = (A^{+}, A^{-}\rangle$ be a bipolar valued Q-fuzzy subgroup of a group G and α_i in $[0,1]$, β_{j} in $[-1,0]$, $\alpha_i \leq A^{+}(e, q)$, $\beta_{\text{j}} \geq A^{-}(e, q)$ where i, j $\in I$. The intersection of a collection of (α, β)-level subgroup of A is also a (α, β)-level subgroup of A.

Theorem 3.10 The homomorphic image of a (α, β)-level subgroup of bipolar valued Q-fuzzy subgroup of a group G is a (α, β)-level subgroup of a bipolar valued Q-fuzzy subgroup of a group G'.

Proof: Let $V = f(A)$. Hence $A = (A^{+}, A^{-}\rangle$ is a bipolar valued Q-fuzzy subgroup of G.

Let $(G,\cdot)$ and $(G',\cdot)$ be any two groups. The homomorphic image of a bipolar valued Q-fuzzy subgroup of G is a bipolar valued Q-fuzzy subgroup of G'. Clearly, $V = (V^{+}, V^{-}\rangle$ is a bipolar valued Q-fuzzy subgroup of G'.

Let x and y in G and q in Q. Then $f(x)$ in G'.

Let $A_{(\alpha,\beta)}$ be a (α, β)-level subgroup of A. That is $A^{+}(x, q) \geq \alpha$ and $A^{-}(x, q) \leq \beta$, $A^{+}(y, q) \geq \alpha$ and $A^{-}(y, q) \leq \beta$; $A^{+}(xy^{-1}, q) \geq \alpha$, $A^{-}(xy^{-1}, q) \geq \beta$.

We have to prove that $f(A_{(\alpha,\beta)})$ is a (α, β)-level subgroup of V. Now, $V^{+}(f(x), q) \geq$

$A^{+}(x, q) \geq \alpha$, which implies that,

$V^{+}(f(x), q) \geq \alpha$ and $V^{+}(f(y), q) \geq A^{+}(y, q) \geq \alpha \Rightarrow V^{+}(f(y), q) \geq \alpha$.

Then $V^{+}(f(x)(f(y)^{-1}, q) = V^{+}(f(xy^{-1}), q) \geq A^{+}(xy^{-1}, q) \geq \alpha$.

Which implies that, $V^{+}(f(x)(f(y)^{-1}, q) \geq \alpha$ and $V^{-}(f(x), q) \leq A^{-}(x, q) \leq \beta$. Which implies that

$V^{-}(f(x), q) \leq \beta$ and $V^{-}(f(y), q) \leq A^{-}(y, q) \leq \beta$.

Which implies that $V^{-}(f(x), q) \leq \beta$.

Then $V^{-}(f(x), (f(y))^{-1}, q) = V^{-}(f(xy^{-1}), q) \leq A^{-}(xy^{-1}, q) \leq \beta$

$\therefore V^{-}(f(x), (f(y))^{-1}, q) \leq \beta$. Hence $f(A_{(\alpha,\beta)})$ is a (α, β)-level subgroup of a bipolar valued Q- fuzzy subgroup of $'$.

Theorem 3.11 Let $A = (A^{+}, A^{-}\rangle$ be a bipolar valued Q- fuzzy subgroup of a group G if any two (α, β)-level subgroups of A belongs to G, then their intersection is also (α, β)-level subgroup of A in G.

Proof: Let α_1, α_2 in $[0,1]$, β_1, β_2 in $[-1,0]$, $\alpha_1 \leq A^{+}(e, q), \alpha_2 \leq A^{+}(e, q), \beta_1 \geq A^{-}(e, q), \beta_2 \geq A^{-}(e, q)$.

Case (i): If $\alpha_1 < A^{+}(x, q) < \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_2,\beta_2)} \subseteq A(\alpha_1, \beta_1)$. Therefore $A(\alpha_1, \beta_1) \cap A(\alpha_2, \beta_2) = A(\alpha_2, \beta_2)$, but $A(\alpha_2, \beta_2)$ is a (α, β)-level subgroup of A.

Case (ii): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A(\alpha_2, \beta_2)$. Therefore $A(\alpha_1, \beta_1) \cap A(\alpha_2, \beta_2) = A(\alpha_1, \beta_1)$, but $A(\alpha_1, \beta_1)$ is a (α, β)-level subgroup of A.

Case (iii): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A(\alpha_1, \beta_2)$. Therefore $A(\alpha_2, \beta_1) \cap A(\alpha_1, \beta_2) = A(\alpha_2, \beta_1)$, but $A(\alpha_2, \beta_1)$ is a (α, β)-level subgroup of A.

Case (iv): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A(\alpha_2, \beta_1)$. Therefore $A(\alpha_1, \beta_2) \cap A(\alpha_2, \beta_1) = A(\alpha_1, \beta_2)$, but $A(\alpha_1, \beta_2)$ is a (α, β)-level subgroup of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_2)} = A(\alpha_2, \beta_2)$.

In all the cases, intersection of any two (α, β)-level subgroups is a (α, β)-level subgroups is a (α, β)- level subgroup of A.

Theorem 3.12 Let $A = (A^{+}, A^{-}\rangle$ be a bipolar valued Q- fuzzy subgroup of a group G if any two (α, β)-level subgroups of A belongs to G, then their union is also (α, β)-level subgroup of A.

Proof: Let α_1, α_2 in $[0,1]$, β_1, β_2 in $[-1,0]$, $\alpha_1 \leq A^{+}(e, q), \alpha_2 \leq A^{+}(e, q), \beta_1 \geq A^{-}(e, q), \beta_2 \geq A^{-}(e, q)$.

Case (i): If $\alpha_1 < A^{+}(x, q) < \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_2,\beta_2)} \subseteq A(\alpha_1, \beta_1)$.

Therefore $A(\alpha_1, \beta_1) \cup A(\alpha_2, \beta_2) = A(\alpha_1, \beta_1)$, but $A(\alpha_1, \beta_1)$ is a (α, β)-level subgroup of A.

Case (ii): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A(\alpha_2, \beta_2)$.

Therefore $A(\alpha_1, \beta_1) \cup A(\alpha_2, \beta_2) = A(\alpha_2, \beta_2)$, but $A(\alpha_2, \beta_2)$ is a (α, β)-level subgroup of A.

Case (iii): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A(\alpha_1, \beta_2)$.

Therefore $A(\alpha_2, \beta_1) \cup A(\alpha_1, \beta_2) = A(\alpha_1, \beta_2)$, but $A(\alpha_1, \beta_2)$ is a (α, β)-level subgroup of A.

Case (iv): If $\alpha_1 > A^{+}(x, q) > \alpha_2$ and $\beta_1 > A^{-}(x, q) > \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A(\alpha_2, \beta_1)$.

Therefore $A(\alpha_1, \beta_2) \cup A(\alpha_2, \beta_1) = A(\alpha_2, \beta_1)$, but $A(\alpha_2, \beta_1)$ is a (α, β)-level subgroup of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A(\alpha_2, \beta_2)$.

In all the cases, union of any two (α, β)-level subgroups is a (α, β)-level subgroups is a (α, β)-level subgroup of A.

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